$\frac{q}{m}$ of Electron

Jeffrey Sharkey, Spring 2006
Phys. 2033: Quantum Lab

1 Purpose

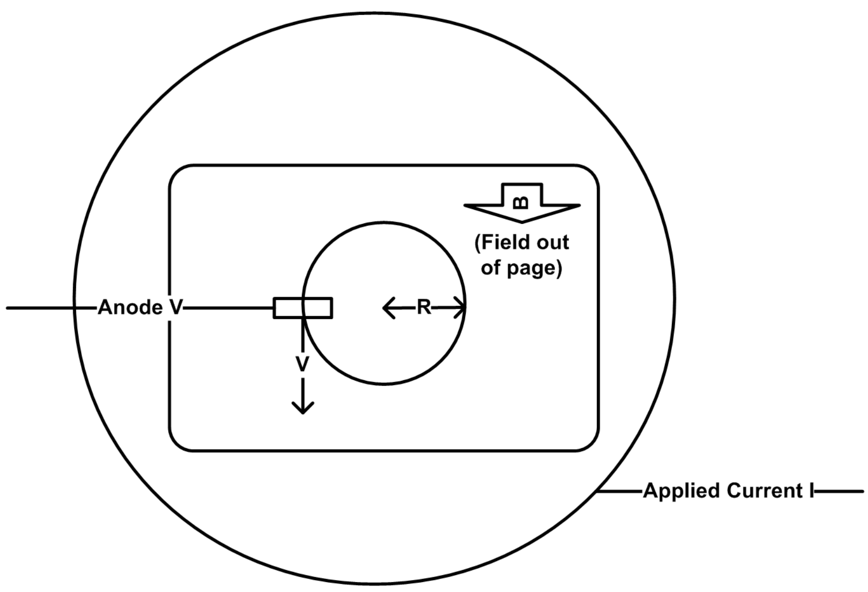
To observe and measure the elementary ratio $\frac{q}{m}$ of electrons.

2 Methodology

By controlling uniform magnetic field, we changed the orbital radius of electrons fired from a cathode. By measuring the field required to hit several targets of a known radius, we can experimentally calculate the $\frac{q}{m}$ ratio.

2.1 Equipment Used

To observe the electron paths, we used an electron gun enclosed in a glass tube filled with Hg vapor. It was operated by applying one voltage to the cathode, and another voltage to the anode. We used a voltmeter to monitor the voltage applied to the anode, and an ammeter to measure the current from electrons captured by the anode.



We also used two large Helmholtz coils to generate and control a uniform magnetic field around the glass tube. Another voltmeter monitored the voltage we applied to the coils.

3 Collected Data

Below is all data collected during the lab.

Trial	Voltage (V)	Current (A)
1	0.0035	0.035
2	0.0068	0.068
3	0.0089	0.089
4	0.0035	0.035
5	0.0042	0.042
6	0.005	0.05
7	0.005	0.05
8	0.0063	0.063

Table 1: Measured applied voltage on Helmholtz coils required to counteract natural magnetic field in room. Current was calculated from known relationship $I = \frac{V}{0.1\Omega}$.

Radius (m)	Voltage (V)	Current (A)
0.057	0.137	1.317
0.051	0.151	1.464
0.045	0.169	1.641
0.039	0.193	1.876
0.032	0.228	2.226
0.057	0.132	1.270
0.051	0.148	1.431
0.045	0.166	1.612
0.039	0.190	1.846
0.032	0.224	2.186
0.057	0.137	1.321
0.051	0.152	1.466
0.045	0.171	1.657
0.039	0.192	1.866
0.032	0.229	2.236

Radius (m)	Voltage (V)	Current (A)
0.057	0.152	1.472
0.051	0.169	1.645
0.045	0.191	1.862
0.039	0.218	2.126
0.032	0.259	2.536
0.057	0.148	1.434
0.051	0.165	1.599
0.045	0.185	1.797
0.039	0.213	2.076
0.032	0.252	2.466
0.057	0.151	1.463
0.051	0.169	1.637
0.045	0.189	1.845
0.039	0.218	2.126
0.032	0.252	2.466

Table 2: Measured applied voltage on Helmholtz coils required to bend electron beam to hit the target at the given radius. The left dataset has anode voltage $\mathbf{V_{app} = 22V}$, while for the right dataset $\mathbf{V_{app} = 28V}$. Shown are data from three separate trials, each group separated by horizontal lines. Current was calculated from known relationship $I = \frac{V}{0.1\Omega} - \overline{I_0}$.

V_{acc}	Radius (m)	$\overline{I} \pm \sigma_I$	$\overline{I_0} \pm \sigma_{I_0}$	$\overline{B}(10^{-4}) \pm \sigma_B$	$\overline{\frac{q}{m}}(10^{11}) \pm \sigma_{q/m}$
22	0.058	1.357 ± 0.028	0.057 ± 0.027	2.550 ± 0.039	2.046 ± 0.039
	0.052	1.508 ± 0.020		2.846 ± 0.033	2.048 ± 0.033
	0.045	1.691 ± 0.023		3.205 ± 0.035	2.115 ± 0.035
	0.039	1.917 ± 0.015		3.649 ± 0.031	2.173 ± 0.031
	0.033	2.270 ± 0.026		4.342 ± 0.037	2.210 ± 0.037
28	0.058	1.456 ± 0.020	0.051 ± 0.009	2.757 ± 0.022	2.229 ± 0.022
	0.052	1.627 ± 0.025		3.091 ± 0.026	2.209 ± 0.026
	0.045	1.835 ± 0.034		3.499 ± 0.035	2.259 ± 0.035
	0.039	2.109 ± 0.029		4.038 ± 0.030	2.258 ± 0.030
	0.033	2.489 ± 0.040		4.783 ± 0.041	2.317 ± 0.041

Table 3: For each anode voltage and radius combination, the average current I and average counteracting I_0. Each σ is calculated from the raw trial data shown in another table. From those values, the average B field and estimated $\frac{q}{m}$ ratio is calculated, along with their corresponding σ errors from earlier fields.

4 Analysis and Results

4.1 Derivation of $\frac{q}{m}$

Electrons in a magnetic field can be accelerated by the force:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

If the electron has an initial velocity perpendicular to the magnetic field, this force simply bends the path into a circular shape with radius r. This centripetal acceleration and radius can be described as:

$$m\frac{v^2}{r} = qvB \tag{2}$$

Which can be rearranged to find our quantity $\frac{q}{m}$:

$$\frac{q}{m} = \frac{v}{Br} \tag{3}$$

In the electron gun, the anode attracts electrons with a voltage V_{acc}. When the electrons reach the anode (or similarly pass through the slit in the anode), they have been accelerated to a kinetic energy:

$$\frac{1}{2}mv^2 = qV_{acc} \tag{4}$$

Rearranging to solve for v, we can substitute into our earlier equation for $\frac{q}{m}$:

$$v = \sqrt{\frac{q}{m}2V_{acc}} \tag{5}$$

$$\frac{q}{m} = \sqrt{\frac{q}{m}2V_{acc}}\frac{1}{Br} \tag{6}$$

We can square both sides, and cancel out one set of the $\frac{q}{m}$ terms, giving us out final equation describing the ratio:

$$\left[\frac{q}{m}\right]^2 = \left[\frac{q}{m}2V_{acc}\right]\frac{1}{(Br)^2} \tag{7}$$

$$\frac{q}{m} = \frac{2V_{acc}}{(Br)^2} \tag{8}$$

We can control the anode voltage and magnetic field, and can measure the radius. Using measured values for each, we can then make an estimate of the ratio $\frac{q}{m}$ for electrons.

4.2 Calculation of B and $\frac{q}{m}$

Thinking of a single charge a distance from a coil, we can find the magnetic field B by integrating around the coil edge. For our solution, x is the distance from the origin of the coil in a perpendicular direction.

$$B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(R^2 + x^2)^{3/2}} \tag{9}$$

When we combine two coils together at a distance R identical to the coil radius, we create a very uniform magnetic field. The x terms in the combined integration are canceled due to the symmetry provided by the coils. This gives us:

$$B = \frac{8\mu_0 NI}{\sqrt{125}R} \tag{10}$$

Actually calculating a value for B where $I = 1.472A$ case, we know that our coil assembly has 72 turns and a radius of $0.33m$.

$$B = \frac{8\mu_0 72(1.472A)}{\sqrt{125}(0.33m)} = 2.887 \times 10^{-4}T \tag{11}$$

Finally, when bringing in our equation for $\frac{q}{m}$ from above, for the case where our curl radius is $0.0575m$ and $V_{acc} = 28V$:

$$\frac{q}{m} = \frac{2(28V)}{(2.887 \times 10^{-4}T)^2(0.0575m)^2} = 2.031 \times 10^{11}\frac{C}{kg} \tag{12}$$

We perform these calculations after averaging the outcomes of several trials and show the results in a table above.

4.3 Earth's Magnetic Field

Looking at out average value for I_0, we can calculate the natural magnetic field present in the lab. From our eight measurements, we found $\sigma = 0.0185$, or an error of about 34.2%.

$$B = \frac{8\mu_0 72(0.054A)}{\sqrt{125}(0.33m)} = 1.059 \times 10^{-5}T \tag{13}$$

The Earth's actual magnetic field is about $3 \times 10^{-5}T$ across most of North America. Our value was of the same order, but $\frac{1}{3}$ smaller. We could explain this difference as experimental error due to the difficult measurements, or we could infer that a positive magnetic field was somewhere above the apparatus, counteracting the Earth's field.

4.4 B Inside the Filament

Ideally, we want only an electric field in the gun. However, the filament current introduces a small B field between the cathode and anode. If no magnetic field were present, we would expect the electrons to spread out in a linear fashion. However, we observed the B field when we noticed that electrons coming from the slit actually *bent* upward and downward; not following the expected linear model.

5 Error Analysis

In this lab, we performed at least three trials of each measurement. This allowed us to calculate the standard deviation σ for each measurement, and to propagate these error estimations through the equations we used. For combining errors, we used the equation introduced in an earlier lab:

$$\sigma_{AB} = \sqrt{\sigma_A^2 + \sigma_B^2} \tag{14}$$

However, in our case we also included the derivative of the equation being used:

$$\partial B = \frac{8\mu_0 72}{\sqrt{125}(0.33m)} = 1.961 \times 10^{-4} \tag{15}$$

$$\partial \frac{q}{m} = \frac{2(22V)}{(1.961 \times 10^{-4})^2 (0.0575m)^2} = 3.457 \times 10^{11} \tag{16}$$

For the averaged values of I, we calculated $\sigma = 0.026$, indicating an average error of 1.441%. Such a low error is very encouraging, indicating that our measurements were performed in a very *uniform* environment. However, for our measurement of the average I_0, we found $\sigma = 0.018$, or a relatively high error of about 31.8%. We can attribute this high error to difficult measurements made with the human eye.

Finally, we compare our average value of $\frac{q}{m} = 2.186 \times 10^{11} \frac{C}{kg}$ to the actual known value $1.76 \times 10^{11} \frac{C}{kg}$. The error compared to this accepted value is 24.2%, which is very high. However, all of our measurements were very uniform, yielding an average combined error of only 3.29% for the $\frac{q}{m}$ ratio. This suggests that any errors we encountered were systematic, and not random.

6 Conclusion

Looking over the entire lab, it was very encouraging to find such uniform results. An error of only 1.441% on average for I, and 3.29% for our final values of $\frac{q}{m}$.

However, we found wildly varying values for the I_0 correcting current, giving it an average error of 31.8%. While the earth's magnetic field was present in the lab, it was still small. In addition, the calibration was done by eye, introducing human error.

Overall, our value of $\frac{q}{m} = 2.186 \times 10^{11} \frac{C}{kg}$ was fairly close to the accepted value. While it was off by about 24.2%, it was of the same 10^{11} order. Given that all our measurements involved a human eye trying to detect the location of a dim light, I was surprised that we ended up so close.